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AdS and pp-wave BPS-particle superalgebras

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Abstract: We examine a limitting procedure of superalgebras for a BPS-particle in $AdS_2 \times S^2$, pp-wave and flat backgrounds focussing on BPS-brane charges. The BPS-particle charge in the flat superalgebra is traced back to a part of Lorentz rotation generators in the AdS and pp-wave superalgebras. A Wess-Zumino term for a BPS-brane causes a momentum shift and the contribution is absorbed into bosonic generators, then it results in closure of the AdS and pp-wave superalgebras.

Keywords: p-branes, D-branes, Penrose limit and pp-wave background, Space-Time Symmetries.

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Contents

JHEP12(2003)016

1. Introduction

Recently brane dynamics in curved backgrounds are examined extensively motivated by the AdS/CFT correspondence [1] and its pp-wave limit [2] and [3]. The study of superstrings in AdS spaces has been started in earlier works [4, 5, 6], and further developed to pp-wave spaces [7] and to D-branes [8, 9]. Branes are characterized by brane charges in superalgebras which are topological [10, 11]. Superalgebras manifest non-perturbative dualities [12] and present solitonic solutions systematically [13]. For a flat background p-brane charges are center [12], and classified by non-trivial elements of the Chevalley-Eilenberg (CE) cohomology [14]. However it is unclear whether these properties are common in curved spaces; any p-form charges (except scalars) can not be centers in these backgrounds, and string charges in AdS space can be given as trivial elements of CE cohomology [15]–[19]. So far a limitting procedure of superalgebras in curved spaces including brane charges has not been discussed.

There are several attempts to evaluate brane charges in superalgebras in the matrix theory approach for pp-wave backgrounds in earlier works [20] and in [21] and in the Green-Schwarz type action approach for the AdS space [19], where new contributions to brane charges are obtained in addition to the flat part. It is desirable to have background dependence of a brane charge, since a brane charge is proportinal to a brane volume which depends on a background. However these brane charges are not allowed by the central extension of the superalgebra. Possible extension of the super-AdS algebra is also discussed [22]. In order to clarify background dependence of a brane charge and consistency of superalgebras, we analyze a simple case and show how these issues incorporate. We use the supermatrix-valued coordinates introduced in "supertwistor formulation" [16] and compute brane charges analogously to [19]. We take a BPS 0-brane case in the $AdS_2\times S^2$ [6] which is the near horizon geometry of the Reissner-Nordström black hole in four dimensions. Its pp-wave limit and a flat limit are examined.

The organization of this paper is as follows. In section 2.1, we explain the coset construction based on the reference [16]. Its Penrose limit [25] is taken for the bosonic part of $AdS_2 \times S^2$ in section 2.2 and for whole part in section 2.3. In section 3 an action for a BPS 0-brane in the $AdS_2 \times S^2$ space is given. The pseudo-supersymmetric WZ term is confirmed to satisfy three criteria; 1. its exterior derivative to be correct two-form field strength, 2. kappa-invariance of the action, 3. correct flat limit. Its pp-wave limit and a flat limit are also given. In section 4 supercharges are obtained and brane charges are calculated by anticommutators of these supercharges.

2. Super-$AdS_2 \times S^2$ and its Penrose limit

Branes in the super-$AdS_2 \times S^2$ background is expressed by the coset

$$\mathrm{SU}(1,1|2)/\,\mathrm{SO}(1,1)\times \mathrm{SO}(2) \longrightarrow \mathrm{GL}(2|2)/(\mathrm{GL}(1))^4\,, \tag{2.1}$$

where Wick rotations are performed in order to use real coset parameters as coordinates. Scaling degrees of freedom $\mathrm{GL}(1)^2$ are introduced in order to describe the system in terms of unconstrained $\mathrm{GL}(2|2)$ variables. An element of the coset is denoted by ${Z_M}^A$ with indices M for the global left $\mathrm{GL}(2|2)$ and indices A for the local right one. Two $\mathrm{GL}(1)$'s are Lorentz groups of two 2-dimensional pseudospheres and other two $\mathrm{GL}(1)$'s are scalings. At first we examine the bosonic part of the $AdS_2 \times S^2$ space, $[\mathrm{GL}(2)/(\mathrm{GL}(1))^2]^2$, in section 2.1 and take the Penrose limit to derive the pp-wave metric in section 2.2. The Penrose limit of the whole supergroup is presented in section 2.3.

2.1 $AdS_2 \times S^2$

The bosonic part of the $AdS_2\times S^2$ space is $[\mathrm{GL}(2)/\,\mathrm{GL}(1)^2]^2$. Coset elements ${z_m}^a$ and ${z_{\bar m}}^{\bar a}$ correspond to a coset $\mathrm{GL}(2)/(\mathrm{GL}(1))^2$ for AdS_2 and another one for S^2 respectively. The left-invariant (LI) one forms are given by

$${j_a}^b = {z_a}^m d{z_m}^b\,,\qquad {j_{\bar a}}^{\bar b} = {z_{\bar a}}^{\bar m} d{z_{\bar m}}^{\bar b}\,, \tag{2.2}$$

where ${z_a}^m$ and ${z_{\bar a}}^{\bar m}$ are inverse of ${z_m}^a$ and ${z_{\bar m}}^{\bar a}$. These indices are contracted with the Lorentz invariant metric δ^{ab} and $\delta^{\bar a\bar b}$ and the Lorentz $\mathrm{GL}(1)$ is assigned to $\mathrm{SO}(2)$. A LI current j_{ab} is decomposed into three parts; a trace part $\delta_{ab}j$, a traceless symmetric part $j_{\langle ab\rangle}$ and an antisymmetric part $j_{[ab]}$,

$$j_{ab} = \frac{1}{2}\delta_{ab}j + j_{\langle ab\rangle} + j_{[ab]}\,. \tag{2.3}$$

If the following parameterization is chosen

$${z_m}^a = R\delta^a_m + \begin{pmatrix} x^0 & x^1 \\ x^1 & -x^0 \end{pmatrix} \tag{2.4}$$

whose inverse is

$$z_a{}^m = \frac{1}{R^2 - (x^0)^2 - (x^1)^2}\left[R\delta_a^m - \begin{pmatrix} x^0 & x^1 \\ x^1 & -x^0 \end{pmatrix}\right], \tag{2.5}$$

then the LI one forms are given by

$$j_{\langle ab\rangle} = \frac{R}{R^2 - \sum_\mu (x^\mu)^2}\begin{pmatrix} dx^0 & dx^1 \\ dx^1 & -dx^0 \end{pmatrix}. \tag{2.6}$$

The metric of this Wick rotated $AdS_2 \times S^2$ is given by

$$\begin{aligned} ds^2_{\text{“AdS}_2\times\text{S}^2\text{”}} &= -\frac{1}{2}\left(j_{\langle ab\rangle}j^{\langle ab\rangle} - j_{\langle\bar a\bar b\rangle}j^{\langle\bar a\bar b\rangle}\right) \\ &= -\frac{R^2}{[R^2 - \sum_\mu (x^\mu)^2]^2}\sum_{\mu=0,1}(dx^\mu)^2 + \frac{R^2}{[R^2 - \sum_\mu (x^\mu)^2]^2}\sum_{\mu=2,3}(dx^\mu)^2 \end{aligned} \tag{2.7}$$

expressing two 2-dimensional pseudospheres with the same unit curvature, where the supersymmetry is imposed to restrict curvatures to be same.

2.2 Penrose limit of $AdS_2 \times S^2$

Under the global GL(2) which is generated by $G_m{}^n$'s

$$G_m{}^n = z_m{}^a \pi_a{}^n\,, \qquad [z_m{}^a, \pi_b{}^n] = \delta_b^a \delta_m^n\,, \qquad \left[G_m{}^n, G_l{}^k\right] = -\delta_l^n G_m{}^k + \delta_m^k G_l{}^n \tag{2.8}$$

with the conjugate momenta π's, a contravariant vector

$$w_{\langle mn\rangle} = z_{\langle m}{}^a z_{n\rangle a} \tag{2.9}$$

is transformed as

$$\delta_\lambda w_{mn} = \left[w_{mn}, \lambda_k{}^l G_l{}^k\right] = \lambda_m{}^l w_{ln} + \lambda_n{}^l w_{lm}\,. \tag{2.10}$$

If we parameterize $z_m{}^a$ as (2.4), $w_{\langle mn\rangle}$ becomes

$$w_{\langle mn\rangle} = 2R\begin{pmatrix} x^0 & x^1 \\ x^1 & -x^0 \end{pmatrix}. \tag{2.11}$$

So the GL(2) generators are parameterized as

$$G^{mn} = \begin{pmatrix} D + P_0 & P_1 + M_{01} \\ P_1 - M_{01} & D - P_0 \end{pmatrix}, \tag{2.12}$$

where D, P, M are GL(1), translations, Lorentz generators respectively. The same parameterization is chosen for another GL(2) generators $G^{\bar m\bar n}$ with replacing P_0, P_1, M_{01} by P_3, P_2, M_{32}.

The Penrose limit is taken as follows: Introduce the lightcone variables and scale them with the parameter Ω then take a limit $\Omega \to 0$

$$\begin{aligned} P_\pm &= \frac{1}{\sqrt{2}}(P_3 \pm P_0) \\ P_+ &\to \Omega^{-2}P_+\,, \quad (P_i,\ M_{01},\ M_{32}) \to \Omega^{-1}(P_i,\ M_{01},\ M_{32})_{\ i=1,2}\,, \quad (P_-,\ D) \to (P_-,\ D) \end{aligned} \tag{2.13}$$

and therefore

$$x^{\pm} = \frac{1}{\sqrt{2}}(x^3 \pm x^0)\,, \qquad x^+ \to \Omega^2 x^+\ ,\ x^i \to \Omega x^i\,, \qquad x^- \to x^-\,. \tag{2.14}$$

The metric (2.7), which is scaled as $ds^2 \to \Omega^2 ds'^2$ and omit prime from now on, becomes

$$\begin{aligned} ds^2_{\rm pp} &= \Omega^{-2}(-\frac{1}{2})[(j^{\langle ab\rangle})^2 - (j^{\langle\bar{a}\bar{b}\rangle})^2]\,|_{\text{rescaling (2.14)},\ \Omega\to 0} \\ &= \frac{R^2}{[R^2 - \frac{1}{2}x^{-2}]^2}\left\{2dx^+dx^- + \sum_{i=1,2}(dx^i)^2 + \frac{2dx^+dx^- + \sum_{i=1,2}(x^i)^2}{R^2 - \frac{1}{2}(x^-)^2}(dx^-)^2\right\} \end{aligned} \tag{2.15}$$

with $x^i = (ix^1, x^2)$. After field redefinition

$$\left\{\begin{aligned} X^i &= \frac{R}{R^2 - \frac{1}{2}(x^-)^2}x^i \\ X^+ &= \frac{R}{R^2 - \frac{1}{2}(x^-)^2}\left(x^+ - \frac{1}{2}\frac{x^-}{R^2 - \frac{1}{2}(x^-)^2}(x^i)^2\right) \\ X^- &= \frac{1}{2}\ln\frac{\sqrt{2}R + x^-}{\sqrt{2}R - x^-} \end{aligned}\right. , \tag{2.16}$$

(2.15) recast into the familiar form

$$ds^2_{\rm pp} = 2dX^+X^- + \sum_{i=1,2}(dX^i)^2 + \frac{2}{R^2}\sum_{i=1,2}(X^i)^2(dX^-)^2\,. \tag{2.17}$$

A flat limit is taken by rescaling $x^\mu \to (1/R)x^\mu$ and $ds^2 \to (1/R^2)ds^2$ then $R \to \infty$. The metric (2.7) has already dilatation scale R exists which is identified to the raduis of the curvature,

$$\begin{aligned} ds^2_{\rm flat} &= R^2\left(-\frac{1}{2}\right)\left[j_{\langle ab\rangle}j^{\langle ab\rangle} - j_{\langle\bar{a}\bar{b}\rangle}j^{\langle\bar{a}\bar{b}\rangle}\right]|_{R\to\infty} \\ &= \left\{-\frac{1}{[1 - \sum_\mu (x^\mu/R)^2]^2}\sum_{\mu=0,1}(dx^\mu)^2 + \frac{1}{[1 - \sum_\mu (x^\mu/R)^2]^2}\sum_{\mu=2,3}(dx^\mu)^2\right\}|_{R\to\infty} \\ &= \sum_{\mu=0,1,2,3}(dx^\mu)^2\,. \end{aligned} \tag{2.18}$$

2.3 Penrose limit of super- $AdS_2 \times S^2$

Now we consider a coset $\mathrm{GL}(2|2)/(\mathrm{GL}(1))^4$ whose element is denoted by $Z_M{}^A$, and the supergroup GL(2|2) is generated by

$$\begin{aligned} G_M{}^N &= Z_M{}^A \Pi_A{}^N\,, \qquad \left[Z_M{}^A, \Pi_B{}^N\right\} = (-1)^A \delta_B^A \delta_M^N \\ \left[G_M{}^N, G_L{}^K\right\} &= (-1)^N\left(-\delta_L^N G_M{}^K + \delta_M^K G_L{}^N\right)\,, \end{aligned} \tag{2.19}$$

where conjugate momenta Π's are introduced. Corresponding to that bosonic lightcone generators are defined as (2.13) for generators of GL(2) in (2.12), fermionic lightcone generators are identified from the above algebra; for example, $\{G_{\bar{1}}{}^1, G_1{}^{\bar{1}}\} \propto P_- \Rightarrow G_1{}^{\bar{1}}, G_{\bar{1}}{}^1 \in Q_-$, $\{G_{\bar{2}}{}^1, G_1{}^{\bar{2}}\} \propto P_+ \Rightarrow G_1{}^{\bar{2}}, G_{\bar{2}}{}^1 \in Q_+$ and so on. Whole identification of lightcone generators with GL(2|2) is given as follows:

$$
G_{MN} = \begin{array}{|c|c|c|c|}\hline D+P_0 & P_1+M_{01} & Q_- & Q_+ \\ \hline P_1-M_{01} & D-P_0 & Q_+ & Q_- \\ \hline Q_- & Q_+ & \bar{D}+P_3 & P_2+M_{32} \\ \hline Q_+ & Q_- & P_2-M_{32} & \bar{D}-P_3 \\ \hline \end{array}
$$

where $P_0 = (P_+ - P_-)/\sqrt{2}$ and $P_3 = (P_+ + P_-)/\sqrt{2}$. The Penrose limit is given by rescaling (2.13) and

$$
Q_+ \to \Omega^{-1} Q_+ \,, \qquad Q_- \to Q_- \,, \tag{2.20}
$$

or equivalently rescaling of GL(2|2) generators as follows

$$
\begin{aligned}
G_{MN} \to \Omega^{-2} & \begin{array}{|c|c|c|c|}\hline P_+ & & & \\ \hline & P_+ & & \\ \hline & & P_+ & \\ \hline & & & P_+ \\ \hline \end{array} + \Omega^{-1} \begin{array}{|c|c|c|c|}\hline & P_1, M & & Q_+ \\ \hline P_1, M & & Q_+ & \\ \hline & Q_+ & & P_2, \bar{M} \\ \hline Q_+ & & P_2, \bar{M} & \\ \hline \end{array} \\
& + \Omega^0 \begin{array}{|c|c|c|c|}\hline D, P_- & & Q_- & \\ \hline & D, P_- & & Q_- \\ \hline Q_- & & \bar{D}, P_- & \\ \hline & Q_- & & \bar{D}, P_- \\ \hline \end{array}
\end{aligned} \tag{2.21}
$$

and taking the limit $\Omega \to 0$.

3. BPS 0-brane actions

The action of a BPS 0-brane in $AdS_2 \times S^2$ is given in [6] and we reformulate this into the "supertwistor formulation" in this section. We must construct the WZ term with taking care of the boundary term to obtain brane charges. In section 3.1 we present a general argument of the pseudo-supersymmetric WZ term [19]. Then the BPS 0-brane action is examined explicitly in section 3.2.

3.1 WZ term for a p-brane

In order to compute a brane charge by the anticommutator of supercharges we need to construct the pseudo-supersymmetric WZ term. For an group element, Z, it can be parameterized generally as $Z = \mathbf{1} + X$ whose inverse is $Z^{-1} = \sum_{n=0}^{\infty} (-X)^n$. A left-invariant (LI) one form current is given as

$$
J = Z^{-1} dZ = \sum_{n=0}^{\infty} (-X)^n \; dX \,, \tag{3.1}
$$

satisfying the Maurer-Cartan equation, $dJ = -JJ$. The right hand side of the integration of the Maurer-Cartan equation is calculated as

$$
-\int JJ = \int d \left(\sum_{n=1}^{\infty} (-X)^n \; dX \right) = \sum_{n=1}^{\infty} (-X)^n \; dX + \text{boundary term}\,. \tag{3.2}
$$

JHEP12(2003)016

Its left-invariance requires "boundary term $= dX$", so "$-\int_{\mathrm{L.I.}} JJ = J$". On the other hand the "pseudo"-left-invariant one form is $-\int_{\mathrm{P.L.I.}} JJ = J + df(X)$ where $f(X)$ is an arbitrary function of X. Among the ambiguity the "correct flat limit" criterion chooses $f(X) = -X$

$$-\int_{\mathrm{P.L.I.}} JJ = J - dX\,. \tag{3.3}$$

For super-p-branes the Wess-Zumino (WZ) term is given as the one-dimensional higher space integration of LI $(p+2)$ form currents

$$\mathcal{L}_{WZ} = B_{[p+1]} = \int_{\mathrm{P.L.I.}} H_{[p+2]}\,, \qquad H_{[p+2]} = J^{p+2}\,, \tag{3.4}$$

which is pseudo-supersymmetry invariant producing the topological p-brane charge. This pseudo-invariance corresponds to the choice of the boundary term of the integration to be zero. If a LI $(p+1)$ form current exists $\tilde{B}_{[p+1]} = J^{p+1}$ with $d\tilde{B}_{[p+1]} = H_{[p+2]}$, the pseudo-invariant $(p+1)$-form is given by [19]

$$\mathcal{L}_{WZ} = B_{[p+1]} = \tilde{B}_{[p+1]} - d\left(X(dX)^p\right). \tag{3.5}$$

The super-AdS group and the nondegenerate supertranslation group are the cases where $\tilde{B}_2 = (J)^2$ exists [15]–[19]. For the nondegenerate supertranslation group there exists the LI 2-form current

$$\tilde{B}_{[2]} = J_{\tilde{Q}} J_Q\,, \qquad d\tilde{B}_{[2]} = H_{[3]} = J_Q J_P J_Q\,, \tag{3.6}$$

where $\tilde{Q}$ is introduced to make a nondegenerate fermionic group metric [24] and satisfies $dJ_{\tilde{Q}} = J_P J_Q$. The WZ term for a superstring in a flat space is given by

$$\mathcal{L}_{WZ} = B_{[2]} = \tilde{B}_{[2]} - d\left(X_{\tilde{Q}} dX_Q\right). \tag{3.7}$$

The auxiliary variable $X_{\tilde{Q}}$ is not contained in the kinetic term, so this can be gauged away in this case.

The WZ term in super-AdS spaces has the form of (3.5) and it must satisfy following three criteria;

1. Its exterior derivative produces the correct field strength, $d\mathcal{L}_{WZ,[p+1]} = H_{[p+2]}$ with $dH_{[p+2]} = 0$.

2. The action is kappa-symmetric.

3. Its flat limit reduces into the correct flat WZ term.

3.2 BPS 0-brane action

Now let us go back to the curved background cases. The supercoset construction is obtained by replacing bosonic currents (2.2) with supercurrents

$$J_A{}^B = Z_A{}^M dZ_M{}^B\,, \qquad Z_M{}^A = \begin{pmatrix} \mathbf{1} & \\ & \mathbf{1} \end{pmatrix} + \begin{pmatrix} X & \Theta \\ \bar{\Theta} & \bar{X} \end{pmatrix}. \tag{3.8}$$

At first all components of X and $\bar{X}$ are used without gauge fixing [16, 18]. Especially the antisymmetric auxiliary variables are important which plays a role of origin of the BPS-particle charge.

The action for a BPS 0-brane is given as

$$
\begin{aligned}
I &= \int d\tau\ (\mathcal{L}_0 + \mathcal{L}_{WZ}) \\
\mathcal{L}_0 &= T\sqrt{g_{00}}\,, \qquad g_{00} = \frac{\left(J_0^{\langle ab\rangle} J_{\langle ab\rangle,0} - J_0^{\langle \bar{a}\bar{b}\rangle} J_{\langle \bar{a}\bar{b}\rangle,0}\right)}{2} \\
\mathcal{L}_{WZ} &= \left\{ K\epsilon_{ab}\left(J_0^{ab} - d(\delta^{am}X_m{}^b)\right) + \bar{K}\epsilon_{\bar{a}\bar{b}}\left(J_0^{\bar{a}\bar{b}} - d(\delta^{\bar{a}\bar{m}}\bar{X}_{\bar{m}}{}^{\bar{b}})\right)\right\},
\end{aligned}
\tag{3.9}
$$

where $J^{AB} = d\tau J_0^{AB}$ and index 0 is omitted from now on. K and $\bar{K}$ are determined by the kappa-symmetry as

$$
T^2 = K^2 + \bar{K}^2\,. \tag{3.10}
$$

This WZ term satisfies three criteria: The criterion 1. is confirmed easier. The two-form field strength for a BPS 0-brane is given by

$$
H_{[2]} = d\mathcal{L}_{WZ} = -\left\{ K\epsilon_{ab}(J^{ac}J_c{}^b + J^{a\bar{c}}J_{\bar{c}}{}^b) + \bar{K}\epsilon_{\bar{a}\bar{b}}(J^{\bar{a}c}J_c{}^{\bar{b}} + J^{\bar{a}\bar{c}}J_{\bar{c}}{}^{\bar{b}})\right\}. \tag{3.11}
$$

The field strength $H_{[2]}$ in a flat space contains bilinear of fermionic indices currents only. On the other hand the one in $\mathrm{AdS}^2\times \mathrm{S}_2$ space contains bilinear of not only the fermionic indices currents but also the bosonic indices currents in order to make closed form $dH_{[2]} = 0$ [6]. For the expression of $\mathcal{L}_{WZ}$ in (3.9) this closure is trivial, i.e. $dH_{[2]} = dd\mathcal{L}_{WZ} = 0 = ddJ$.

The criterion 2. is calculated as follows: If we introduce the following combination for an arbitrary variation $\delta Z_M{}^A$

$$
\Delta J^{AB} = Z^{AM}\delta Z_M{}^B\,, \tag{3.12}
$$

the kappa-variation is characterized by vanishing bosonic components

$$
\Delta_{\mathcal{K}} J^{ab} = 0 = \Delta_{\mathcal{K}} J^{\bar{a}\bar{b}} \tag{3.13}
$$

as usual. In particular in this parameterization $\Delta_{\mathcal{K}} J^{a\bar{b}} = \mathcal{K}^{a\bar{b}}$ and $\Delta_{\mathcal{K}} J^{\bar{a}b} = \mathcal{K}^{\bar{a}b}$.[1] Using the kappa-variation of currents,

$$
\delta_{\mathcal{K}} J^{ab} = J^{a\bar{c}}\Delta_{\mathcal{K}} J_{\bar{c}}{}^b - (\Delta_{\mathcal{K}} J^{a\bar{c}})J_{\bar{c}}{}^b \tag{3.16}
$$

[1] In the canonical computation the fermionic constraints are given as [18]

$$
\tilde{D}_{\bar{a}b} = D_{\bar{a}b} + \frac{\partial\mathcal{L}_{WZ}}{\partial\dot{\Theta}_m{}^{\bar{a}}}X_m{}^b = 0\,, \qquad \tilde{D}_{a\bar{b}} = D_{a\bar{b}} + \frac{\partial\mathcal{L}_{WZ}}{\partial\dot{\bar{\Theta}}_{\bar{m}}{}^a}\bar{X}_{\bar{m}}{}^{\bar{b}} = 0 \tag{3.14}
$$

with the local GL(2|2) generators, D_{AB}'s. The kappa-variation is generated by these fermionic constraints

$$
\delta_{\mathcal{K}}\mathcal{O} = \left[\mathcal{O}, F_{\bar{a}b}\mathcal{K}^{b\bar{a}} + F_{a\bar{b}}\mathcal{K}^{\bar{b}a}\right], \qquad \delta_{\mathcal{K}} Z_M{}^b = Z_M{}^{\bar{a}}\mathcal{K}_{\bar{a}}{}^b\,, \qquad \delta_{\mathcal{K}} Z_M{}^{\bar{b}} = Z_M{}^a\mathcal{K}_a{}^{\bar{b}} \tag{3.15}
$$

with these parameters are projected into half.

JHEP12(2003)016

and the similar for the $J^{\bar{a}\bar{b}}$, the kappa-variation of the BPS 0-brane action (3.9) becomes

$$\delta_{\mathcal{K}}\mathcal{L}_0 = \frac{T}{\sqrt{g_{00}}}\left\{\Delta_{\mathcal{K}}J_{a\bar{b}}\ \left(J^{ac}J^{\bar{b}}{}_c - J^{\bar{c}\bar{b}}J_{\bar{c}}{}^a\right) + \Delta_{\mathcal{K}}J_{\bar{a}b}\ \left(J^{cb}J_c{}^{\bar{a}} - J^{\bar{a}\bar{b}}J^b{}_{\bar{b}}\right)\right\} \tag{3.17}$$

$$\delta_{\mathcal{K}}\mathcal{L}_{WZ} = -\Delta_{\mathcal{K}}J_{a\bar{b}}\ \left(K\epsilon^{ab}J^{\bar{b}}{}_b + \bar{K}\epsilon^{\bar{a}\bar{b}}J_{\bar{a}}{}^a\right) - \Delta_{\mathcal{K}}J_{\bar{a}b}\ \left(K\epsilon^{ab}J_a{}^{\bar{a}} + \bar{K}\epsilon^{\bar{a}\bar{b}}J^b{}_{\bar{b}}\right). \tag{3.18}$$

The kappa-invariance of the action leads to

$$\begin{aligned}
\delta_{\mathcal{K}}(\mathcal{L}_0 + \mathcal{L}_{WZ}) &= (\Delta_{\mathcal{K}}J_{a\bar{b}}J_{\bar{c}d} + \Delta_{\mathcal{K}}J_{\bar{c}d}J_{a\bar{b}})\mathcal{P}^{a\bar{b}\bar{c}d} \\
\mathcal{P}^{a\bar{b}\bar{c}d} &= \frac{T}{\sqrt{g_{00}}}(J^{ad}\delta^{\bar{b}\bar{c}} - \delta^{ad}J^{\bar{c}\bar{b}}) - (K\epsilon^{ad}\delta^{\bar{b}\bar{c}} + \bar{K}\delta^{ad}\epsilon^{\bar{c}\bar{b}}) \\
0 &= \mathcal{P}^{a\bar{b}\bar{c}d}\Delta_{\mathcal{K}}J_{a\bar{b}} = \mathcal{P}^{a\bar{b}\bar{c}d}\Delta_{\mathcal{K}}J_{\bar{c}d}\,.
\end{aligned} \tag{3.19}$$

Equivalently projected kappa-parameters $\mathcal{P}^{a\bar{b}\bar{c}d}\mathcal{K}_{a\bar{b}} = 0 = \mathcal{P}^{a\bar{b}\bar{c}d}\mathcal{K}_{\bar{c}d}$ with the relation (3.10).

The criterion 3. is evaluated as follows: Under the flat limit which is obtained by rescaling, $X, \bar{X} \to X/R, \bar{X}/R$, $\Theta, \bar{\Theta} \to \Theta/\sqrt{R}, \bar{\Theta}/\sqrt{R}$ and $\mathcal{L}_{WZ} \to \mathcal{L}_{WZ}/R$ as well as $\mathcal{L}_0 \to \mathcal{L}_0/R$, and $R \to \infty$, the WZ term (3.9) reduces into :

$$\mathcal{L}_{WZ,\text{flat}} = -K\left\{\epsilon_{ab}\bar{\Theta}^{\bar{m}a}d\bar{\Theta}_{\bar{m}}{}^b + \epsilon_{\bar{a}\bar{b}}\Theta^{m\bar{a}}d\Theta_m{}^{\bar{b}}\right\}, \tag{3.20}$$

where $K = \bar{K}$ is imposed by requiring 4-dimensional covariance and this is consistent with [26, 27].

The WZ term for a BPS 0-brane in the pp-wave background is obtained by taking the Penrose limit [25] given in (2.21). The WZ term is scaled under the Penrose limit as $\mathcal{L}_{WZ} \to \Omega\mathcal{L}_{WZ}$ as well as the kinetic term $\mathcal{L}_0 \to \Omega\mathcal{L}_0$. Since the antisymmetric components of currents and parameters are scaled as $(J_{[ab]}, J_{[\bar{a}\bar{b}]}) \to \Omega(J_{[ab]}, J_{[\bar{a}\bar{b}]})$, $([X], [\bar{X}]) \to \Omega([X], [\bar{X}])$, all terms in $\mathcal{L}_{WZ}$ are survived. The LI current after taking the Penrose limit, which is still complicated function of $x^- = x_+$ and θ^-, is denoted as $J_{\text{pp},AB}$. Then the WZ term is written as

$$\mathcal{L}_{WZ,\text{pp}} = \left\{K\epsilon_{ab}\left(J^{ab}_{\text{pp}} - d(\delta^{am}X_m{}^b)\right) + \bar{K}\epsilon_{\bar{a}\bar{b}}\left(J^{\bar{a}\bar{b}}_{\text{pp}} - d(\delta^{\bar{a}\bar{m}}\bar{X}_{\bar{m}}{}^{\bar{b}})\right)\right\}. \tag{3.21}$$

4. BPS 0-brane charges

The global GL(2|2) symmetry generators with parameters Λ^{MN} are modified under the existence of the WZ term [11]

$$\Lambda^{NM}G_{MN} \ \to\ \Lambda^{NM}\tilde{G}_{MN} = \Lambda^{NM}G_{MN} - U_\Lambda\,, \qquad \delta_\Lambda\mathcal{L} = dU_\Lambda\,. \tag{4.1}$$

Because of $\delta_\Lambda\mathcal{L}_0 = 0 = \delta_\Lambda J$, only the total derivative terms in the WZ term in (3.9) contribute to U which is given as

$$\begin{aligned}
U_\Lambda &= -\left(K\epsilon_{ab}\delta^{am}\delta_\Lambda X_m{}^b + \bar{K}\epsilon_{\bar{a}\bar{b}}\delta^{\bar{a}\bar{m}}\delta_\Lambda\bar{X}_{\bar{m}}{}^{\bar{b}}\right), \\
\delta_\Lambda Z_M{}^A &= \Lambda_M{}^L Z_L{}^A\,.
\end{aligned} \tag{4.2}$$

This modified GL(2|2) generators are then expressed as

$$
\begin{aligned}
&\tilde{G}_{MN} = G_{MN} + \mathcal{T}_{MN} \\
&\left\{\begin{array}{ll}
\mathcal{T}_{mn} = -K(\delta_m^a + X_m{}^a)\epsilon_{ab}\delta_n^b \ , & \mathcal{T}_{m\bar{n}} = -\bar{K}\Theta_m{}^{\bar{a}}\epsilon_{\bar{a}\bar{b}}\delta_{\bar{n}}^{\bar{b}} \\
\mathcal{T}_{\bar{m}\bar{n}} = -\bar{K}(\delta_{\bar{m}}^{\bar{a}} + \bar{X}_{\bar{m}}{}^{\bar{a}})\epsilon_{\bar{a}\bar{b}}\delta_{\bar{n}}^{\bar{b}} \ , & \mathcal{T}_{\bar{m}n} = -K\bar{\Theta}_{\bar{m}}{}^a\epsilon_{ab}\delta_n^b
\end{array}\right. ,
\end{aligned}
\tag{4.3}
$$

and also re-expressed as

$$
\begin{aligned}
&\tilde{G}_{MN} = Z_M{}^A\tilde{\Pi}_{AN} \\
&\left\{\begin{array}{ll}
\tilde{\Pi}_{an} = \Pi_{an} - K\epsilon_{ab}\delta_n^b \ , & \tilde{\Pi}_{a\bar{n}} = \Pi_{a\bar{n}} \\
\tilde{\Pi}_{\bar{a}\bar{n}} = \Pi_{\bar{a}\bar{n}} - \bar{K}\epsilon_{\bar{a}\bar{b}}\delta_{\bar{n}}^{\bar{b}} \ , & \tilde{\Pi}_{\bar{a}n} = \Pi_{\bar{a}n}
\end{array}\right.
\end{aligned}
$$

satisfying the same algebra of GL(2|2).[2] Despite of the existence of the WZ term the global charges $\tilde{G}_{MN}$ are conserved and generate the closed superalgebra. Fermionic parts are identified as supercharges as $\tilde{G}_{m\bar{n}} \equiv Q_{\mathrm{AdS},m\bar{n}}$, $\tilde{G}_{\bar{m}n} \equiv \bar{Q}_{\mathrm{AdS},\bar{m}n}$, while bosonic parts are interpreted as sum of the bosonic charges G_{MN} and topological charges $\mathcal{T}_{mn}$, $\mathcal{T}_{\bar{m}\bar{n}}$ corresponding to the flat case. Therefore commutators of supercharges for a BPS 0-brane in the $AdS_2 \times S^2$ background are given by

$$
\begin{aligned}
&\{Q_{\mathrm{AdS}}, Q_{\mathrm{AdS}}\} = 0 = \{\bar{Q}_{\mathrm{AdS}}, \bar{Q}_{\mathrm{AdS}}\} \\
&\left\{\bar{Q}_{\mathrm{AdS},\bar{m}}{}^n, Q_{\mathrm{AdS},l}{}^{\bar{k}}\right\} = \\
&\quad = -\delta_l^n(P_{\mathrm{AdS}} + M_{\mathrm{AdS}} + \mathcal{T}_{\mathrm{AdS}})_{\bar{m}}{}^{\bar{k}} + \delta_{\bar{m}}^{\bar{k}}(P_{\mathrm{AdS}} + M_{\mathrm{AdS}} + \mathcal{T}_{\mathrm{AdS}})_l{}^n + \delta_l^n\delta_{\bar{m}}^{\bar{k}}(\bar{D} - D) \ ,
\end{aligned}
\tag{4.4}
$$

where a center $\bar{D} - D = \mathrm{Str}G_{MN}$ can be set to be zero. P_{AdS}'s are symmetric traceless part of bosonic generators G, M_{AdS}'s are antisymmetric part of bosonic generators G and $\mathcal{T}_{\mathrm{AdS}}$'s are bosonic part of (4.3). They have position dependence so they are not center.

Under the Penrose limit (2.21) contributions from the WZ term are given as

$$
\left\{\begin{array}{ll}
\mathcal{T}_{\mathrm{pp},mn} = -K(\mathbf{1} + X\mid_{x^+=0})_m{}^a\epsilon_{ab}\delta_n^b \ , & \mathcal{T}_{\mathrm{pp},m\bar{n}} = -\bar{K}\Theta_m{}^{\bar{a}}\epsilon_{\bar{a}\bar{b}}\delta_{\bar{n}}^{\bar{b}} \\
\mathcal{T}_{\mathrm{pp},\bar{m}\bar{n}} = -\bar{K}(\mathbf{1} + \bar{X}\mid_{x^+=0})_{\bar{m}}{}^{\bar{a}}\epsilon_{\bar{a}\bar{b}}\delta_{\bar{n}}^{\bar{b}} \ , & \mathcal{T}_{\mathrm{pp},\bar{m}n} = -K\bar{\Theta}_{\bar{m}}{}^a\epsilon_{ab}\delta_n^b
\end{array}\right. .
\tag{4.5}
$$

The concrete expression of commutators of supercharges is given by

$$
\left\{\begin{aligned}
&\left\{\bar{Q}_{\mathrm{pp},+,\bar{m}}{}^n, Q_{\mathrm{pp},+,l}{}^{\bar{k}}\right\} = -\delta_l^n\begin{pmatrix}\sqrt{2}P_+ & \\ & -\sqrt{2}P_+\end{pmatrix}_{\bar{m}}{}^{\bar{k}} \\
&\left\{\bar{Q}_{\mathrm{pp},-,\bar{m}}{}^n, Q_{\mathrm{pp},-,l}{}^{\bar{k}}\right\} = -\delta_l^n\left\{\begin{pmatrix}\sqrt{2}P_- & \\ & -\sqrt{2}P_-\end{pmatrix} - \right. \\
&\qquad\qquad \left. - \begin{pmatrix} & Kx^1 - \bar{K}x^2 \\ Kx^1 - \bar{K}x^2 & \end{pmatrix}\begin{pmatrix} & 1 \\ -1 & \end{pmatrix}\right\}_{\bar{m}}{}^{\bar{k}} + \\
&\qquad\qquad + \delta_l^n\delta_{\bar{m}}^{\bar{k}}(D - \bar{D}) \\
&\left\{\bar{Q}_{\mathrm{pp},+,\bar{m}}{}^n, Q_{\mathrm{pp},-,l}{}^{\bar{k}}\right\} = \\
&\quad = -\delta_l^n\left\{\begin{pmatrix} & P_2 + M_{32} \\ P_2 - M_{32} & \end{pmatrix} - \bar{K}\begin{pmatrix}1 + \frac{x^-}{\sqrt{2}} & \\ & 1 - \frac{x^-}{\sqrt{2}}\end{pmatrix}\begin{pmatrix} & 1 \\ -1 & \end{pmatrix}\right\}_{\bar{m}}{}^{\bar{k}} + \\
&\qquad + \delta_{\bar{m}}{}^{\bar{k}}\left\{\begin{pmatrix} & P_1 + M_{01} \\ P_1 - M_{01} & \end{pmatrix} - K\begin{pmatrix}1 - \frac{x^-}{\sqrt{2}} & \\ & 1 + \frac{x^-}{\sqrt{2}}\end{pmatrix}\begin{pmatrix} & 1 \\ -1 & \end{pmatrix}\right\}_l{}^n .
\end{aligned}\right.
\tag{4.6}
$$

[2] We thank Kiyoshi Kamimura for pointing out that a total derivative term does not change the global symmetry algebra.

The anticommutator $\{\bar{Q}_{\text{pp},-,\bar{m}}{}^{n}, Q_{\text{pp},+,l}{}^{\bar{k}}\}$ satisfies the same relation with the last equation of (4.6) with opposite lightcone projection. This is the anticommutators of supercharges for a BPS 0-brane in the 2+2 dimensional pp-wave background, where the super-pp-algebra part is consistent with [2, 23] by setting a center $D-\bar{D} = \text{Str}G_{MN}$ to be zero. The BPS 0-brane charge in the pp-wave background is independent on x^+ but depend on x^i's $(i = 1,2)$ and x^- $(x^- = x_+ = p_+\tau$ in the lightcone gauge). In the lightcone gauge all supersymmetries are broken except $x^i = 0$, since Q_+ are broken because of $P_+ \neq 0$ and Q_- are also broken for $x^i \neq 0$ [8]. It is stressed that this observation is on the coordinate system (2.7), and this coordinates dependence looks differently in the other coordinate systems.

Taking a flat limit leads to the flat superalgebra with the form (4.4) replacing the the indices "AdS" to "flat" where

$$\begin{cases} P_{\text{flat},mn} = \delta^a_m \Pi_{an}\;, & P_{\text{flat},\bar{m}\bar{n}} = \delta^{\bar{a}}_{\bar{m}} \Pi_{\bar{a}\bar{n}} \\ \mathcal{T}_{\text{flat},mn=} - K\delta^a_m \epsilon_{ab}\delta^b_n\;, & \mathcal{T}_{\text{flat},\bar{m}\bar{n}} = -\bar{K}\delta^{\bar{a}}_{\bar{m}}\epsilon_{\bar{a}\bar{b}}\delta^{\bar{b}}_{\bar{n}} \end{cases} \tag{4.7}$$

$$\begin{cases} Q_{\text{flat},m\bar{n}} = \delta^a_m \Pi_{a\bar{n}} + \Theta_m{}^{\bar{a}}\Pi_{\bar{a}\bar{n}} + (-\bar{K}\Theta^{\bar{a}}_m \epsilon_{\bar{a}\bar{b}}\delta^{\bar{b}}_{\bar{n}}) \\ \bar{Q}_{\text{flat},\bar{m}n} = \delta^{\bar{a}}_{\bar{m}} \Pi_{\bar{a}n} + \Theta_{\bar{m}}{}^{a}\Pi_{an} + (-K\Theta^{a}_{\bar{m}} \epsilon_{ab}\delta^{b}_{n}) \end{cases} \tag{4.8}$$

and M's vanish under the flat limit. Imposing four-dimensional symmetry leads to $K = \bar{K}$. Since $\mathcal{T}_{\text{flat}}$'s are anti-symmetric, it can be traced back to a part of Lorentz generators M's in the AdS or pp-wave superalgebras. By taking square of the right hand side of the flat superalgebra and its trace, the BPS condition is obtained as

$$P_0^2 \geq K^2 + \bar{K}^2\,, \tag{4.9}$$

where P_0 is the BPS mass which is well defined after 4-dimensional Lorentz symmetry is restored.

5. Conclusions

We have examined the BPS-particle charges in superalgebras during the limiting procedure among AdS, pp-wave and flat spaces. The BPS-particle charges in the flat-space superalgebra can be traced into the AdS and pp-wave superalgebras; it is absorbed into the Lorentz rotation generators in the AdS and pp-wave superalgebras, and the isometry algebras stay the same keeping the closure of algebras.

In general the BPS p-brane charges in AdS and pp-wave space superalgebras, which reduce to correct ones in the flat limit, are not center. They are rather absorbed into original bosonic generators and avoid the central extension resulting in closure of the same algebra. Therefore WZ term can be given in terms of the trivial class of the CE cohomology of the curved superspace. The WZ term does not modify the original global superalgebra, but it modifies the local symmetry constraint algebra as shown in [18]. This is the same case of the closed string in flat space which has no boundary contribution. The isometry algebras for the super-AdS and pp-wave spaces require the absorption of the boundary contributions.

It is also important to examine general brane configurations. This background covariant approach presents more general description including a BPS 0-brane, which is excluded in the lightcone gauge approach since the lightcone gauge restricts $x^{\pm}$ to be Neumann boundary condition. Toward better understanding of branes, dualities, gauge theory correspondence and quantum theories further studies are required.

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JHEP12(2003)016

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